Golden-ratio search

1. Starting with the interval [4, 5], approximate a minimum of $f(x) \overset{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ with three steps of the golden-ratio search.

Answer: The least values are indicated in bold.

[4, 4.381966011, **4.618033989**, 5]

[4.381966011, **4.618033989**, 4.763932023, 5]

[4.381966011, **4.527864045**, 4.618033989, 4.763932023]

[4.381966011, 4.472135955, **4.527864045**, 4.618033989]

2. The actual minimum is at *x* = 4.542956187. How does the error in the *x*-value and the error in the *f*-value differ with the approximation *x* = 4.527864045?

Answer: The error in the *x*-value is 0.01509, but the *f* (4.527864045) = –0.2061199185 and the value of the function at the actual minimum is –0.2063270794, and the error here is only 0.00002545. Thus, we do not need to be as close to a minimum to actually have an accurate approximation as to what the minimum is.

3. Starting with the interval [– 2, –1.5], approximate a minimum of $f(x) \overset{\text{def}}{=} x^4 - 6x^2 + 4x + 4$ with three steps of the golden-ratio search.

Answer: The least values are indicated in bold.

[–2, **–1.809016994**, –1.690983006, –1.5]

[–2, **–1.881966011**, –1.809016994, –1.690983006]

[–2, –1.927050983, **–1.881966011**, –1.809016994]

[–1.927050983, **–1.881966011**, –1.854101966, –1.809016994]

4. The actual minimum is at *x* = –1.879385242. How does the error in the *x*-value and the error in the *f*-value differ with the approximation *x* = –1.881966011?

Answer: The error in the *x*-value is 0.002581, but the *f* (–1.879385242) = –12.234321066 and the value of the function at the actual minimum is –12.23442238, and the error here is only 0.0001013.

5. Suppose that the width of the interval is initially $w = b_0 - a_0$, but you want the maximum error in *x* to be no more than $\varepsilon_{\text{step}}$. Can you come up with a formula to determine the number of steps that must be taken?

Answer: We need that $\varepsilon_{\text{step}}\, \phi^n \geq w$, so $\phi^n \geq w/\varepsilon_{\text{step}}$ and so $n \geq \log_\phi(w/\varepsilon_{\text{step}})$, so $n = \lceil \log_\phi(w/\varepsilon_{\text{step}}) \rceil$